Mathematics
Grade 12
Discrete Mathematics with Applications
10/31/18
N. Mahoney, K. Boltzer, L. Bylinski, J. Wilke

Compliance Update Completed June 2022

## Wayne School District Curriculum Format

| Content Area/ Grade Level/ Course: | Mathematics Grades 12 Discrete Mathematics with Applications |
| :---: | :---: |
| Unit Plan Title: | Set Theory and Rules of Probability |
| Time Frame | 6 weeks |
| Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| Mathematics: <br> Statistics and Prob <br> S-CP:Conditional <br> S-MD: Using Prob <br> Algebra <br> A-CED: Creating E <br> Standards for Ma <br> MP: Standards 1-6 | lity <br> ability and the Rules of Probability ty to Make Decisions <br> tions <br> ractice |
| Unit Overview |  |
| Statistical reasoning is an essential skill in the information age. It provides the framework for students to evaluate large amounts of data encountered daily and helps to make informed decisions. In this section of the unit, students expand their statistical literacy and analytical thinking. Students become analytical in how they look at data as they develop conceptual understanding and procedural knowledge. <br> In previous courses, students have organized, displayed, and analyzed data and have used displays to identify trends and draw conclusions. In this unit, students expand upon their understanding of data analysis by studying real-world data displays to examine statistical intent. They look at techniques used by statisticians to influence the way data is interpreted. These techniques include: drawing different samples from a population, using different measures of central tendency, and using a variety of displays. <br> The study of set theory extends and develops students' ability to reason mathematically. By identifying sub-groups which share one or more common characteristics, set theory provides a context in which one can examine, compare, and describe relationships of sets. Students develop a common language for set theory and use multiple representations, including Venn diagrams, to solve problems. <br> Students build on their knowledge of mathematical relationships and use authentic situations to investigate set relationships and to formalize vocabulary. This understanding is applied to number lines, rosters, and set builder notation. Students explore Venn diagrams and applications in solving problems, determining probability, and illustrating relationships among number systems. <br> Set theory and statistical applications provide a conceptual framework for examining other topics in this course. These topics empower students to question the world around them, critically analyze information, recognize relationships, and communicate mathematically. This instructional unit on sets provide a step-by-step introduction to sets and set theory. Basic definitions and notation, types of sets, equality, and Venn Diagrams are presented. This unit also covers subsets, the Universal set, set-builder notation, complement, intersection and union. A solid foundation on sets is provided for students of all ages. Connections to other disciplines and to the real world are made throughout. |  |
| Standard Number(s) * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4. |  |
| S-CP:Conditional | bability and the Rules of Probability |

## A. Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of $A$ given $B$ as $P(A$ and $B) / P(B)$, and interpret independence of $A$ and $B$ as saying that the conditional probability of $A$ given $B$ is the same as the probability of $A$, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model
6. Find the conditional probability of $A$ given $B$ as the fraction of $B^{\prime}$ s outcomes that also belong to $A$, and interpret the answer in terms of the model.
7. Apply the Addition Rule, $P(A$ or $B)=P(A)+P(B)-P(A$ and $B)$, and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A$ and $B)=P(A) P(B \mid A)=P(B) P(A \mid B)$, and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

## S-MD: Using Probability to Make Decisions

## A. Calculate expected values and use them to solve problems

1. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
2. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.

## A-CED: Creating Equations

## A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

## MP: Standards for Math Practice

1. Make sense of problems and persevere in solving them.
2. Construct viable arguments and critique the reasoning of others.
3. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Attend to precision
6. Use appropriate tools strategically
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
9.4.12.Cl.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

## Intended Outcomes - \{Essential Questions\}

- Why is writing sets in set builder notation helpful?
- When would roster form be more helpful than set builder notation?
- How is a proper set different from a subset?
- How are sets and their relationships used to calculate probabilities?
- Why are Venn diagrams helpful when illustrating set operations?
- How are sets and Venn diagrams related?
- How can understanding set theory help with prior knowledge of numerical sets?
- Where are sets prevalent in society?
- What are infinite sets and why are they important?


## Enduring Understandings

- Putting elements into sets helps order and arrange the world around us.
- Graphical representations and statistical measures influence interpretations and predictions about data.
- Relationships can be represented using set theory.
- Reading, understanding, interpreting, and communicating data are critical in modeling a variety of realworld situations, drawing appropriate inferences, making informed decisions, and justifying those decisions.
- Probability quantifies the likelihood that something will happen and enables us to make predictions and informed decisions.
- Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting noncontinuous data.
- Discussing and determining the likelihood of an event relies on recognizing when to utilize the fundamental counting principal and recognizing which type of probability we are working with.

In this unit plan, the following $\mathbf{2 1}^{\text {st }}$ Century themes and skills are addressed.


|  | section of the spinner. Have them define two events based on the spinners. For example, if letters are used, the set of vowels and the set of letters in the word math. Ask students to find the probability of each event, their complements, their union, and their intersection. Have students share their work. Review which events have a probability of 1 , which have a probability of 0 , and why. <br> - Connect Vocabulary - Have students create a set of cards with diagrams to help them become familiar with the vocabulary introduced in this lesson. Help students connect the vocabulary to the notation used to represent a set, an element, the universal set, a subset, union, intersection, and complement. Have students use different colors to highlight and distinguish each relationship. <br> - Extension Activity - Have students draw graphs showing "fuzzy"(not binary) sets ranging from cold to hot. (Three sets could show cold, warm, and hot. Four sets could show cold, cool, warm, and hot. However, leave the choice of adjectives and the number of sets to students.) The vertical axis should record degrees of membership from 0 to 1 . The horizontal axis should show either Fahrenheit or Celsius temperatures. Encourage students to be creative with their graphs, for example, by using colors to distinguish sets from one another. Students should write and answer at least three questions involving unions, intersections, and complements of the sets they have graphed. |
| :---: | :---: |
| Differentiation Strategies | Cooperative Learning Structures <br> Multiple Intelligence Activities <br> Performance Assessments <br> Digital Assessment <br> Manipulatives <br> Encourage students to design their own experiments to illustrate what they have learned about probability, such as calculating the complement of rolling a number with a number cube and then attempting to conform the calculation experimentally. Invite students to demonstrate their experiments before the class. <br> Visual Cues <br> When students create Venn diagrams to model a sample space and sets, caution them to be sure that an element is not used more than once on the diagram. For example, have students check that a number does not appear both in Set A and in its intersection with Set B. <br> Journal <br> Have students write and solve their own probability problems. Remind students to use set notation in their solutions to the problems. |
| Resources |  |
| - Textbook <br> - Websites: <br> Explore probability models - https://www.youtube.com/watch?v=fZwKpkhyl_k https://www.youtube.com/watch?v=_vILIwhaqAQ <br> https://www.youtube.com/watch?v=JNm3M9cqWyc <br> https://www.youtube.com/watch?v=se_VxWxilfc |  |

- Calculators
- Technology: Powerpoint, Excel, etc.


## Wayne School District Curriculum Format

| Content Area/ <br> Grade Level/ <br> Course: | Mathematics <br> Grades 12 <br> Discrete Mathematics with Applications |
| :--- | :--- |
| Unit Plan Title: | Logic |
| Time Frame | 5 weeks |
| Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| Mathematics: <br> Statistics and Probability |  |
| S-CP:Conditional Probability and the Rules of Probability |  |
| Standards for Math Practice |  |
| MP: Standards 1,3, and 6 |  |
| Unit Overview |  |
| Logic is a large part of mathematics as well as many other disciplines. Understanding logic will enable a person to |  |
| think clearly, create convincing arguments, and develop patterns of reasoning that will help in making decisions. In |  |
| reading, writing, and speaking, we use words such as "and", "or", and "if...then" to connect thoughts and ideas we are |  |
| trying to get across to another person. In logic we call these words connectives. In daily conversations, these words can |  |
| be interpreted many ways. Statements in the English language are often made up of these connectives, which we will |  |
| notice throughout the unit. Students will use truth tables to illustrate the possible outcomes of a scenario. The truth |  |

table contains the truth values that would occur under the premises of a given scenario. As a result, the table helps visualize whether an argument is logical (true) in a given scenario.

## Standard Number(s) * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

## S-CP:Conditional Probability and the Rules of Probability

## A. Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

## MP: Standards for Math Practice

1. Make sense of problems and persevere in solving them.
2. Construct viable arguments and critique the reasoning of others.
3. Attend to precision.
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
9.4.12.CI.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

## Intended Outcomes - \{Essential Questions\}

- What are the symbols used in logic?
- How can the truth value of a proposition or set of propositions be determined?
- What makes a proof or an argument logically valid
- Is there a best way to write a logically valid proof?
- How is a truth table constructed and used for?
- When is a statement true or false and how can you determine it?
- When is an argument deemed valid and under what conditions is this true?
- How can you tell if two statements are logically equivalent?


## Enduring Understandings

- There is more than one correct way to write logical proof.
- Logical proofs carry more validity than baseless arguments.
- Common language is often riddled with fallacies (sometimes when we communicate, our assumed meaning is not necessarily the logical meaning).
- Use inductive and deductive reasoning to reach conclusions, identify conjectures and counterexamples, and describe the nature of a deductive mathematical system.
- Recognize valid deductive reasoning.
- Constructs and uses proper if-then, converse, inverse, and contrapositive statements.
- Use formal and/or informal logical reasoning processes to make realworld decisions.

In this unit plan, the following $21^{\text {st }}$ Century themes and skills are addressed.

| Check all that apply. 21 ${ }^{\text {st }}$ Century Themes |  | Indicate whether these skills are E-Encouraged, $\boldsymbol{T}$-Taught, or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill. <br> $21^{\text {st }}$ Century Skills |  |
| :---: | :---: | :---: | :---: |
| x | Global Awareness <br> Environmental Literacy <br> Health Literacy <br> Civic Literacy <br> Financial, Economic, Business, and Entrepreneurial Literacy | E,A | Creativity and Innovation <br> Critical Thinking and Problem Solving <br> Communication <br> Collaboration |
|  |  | E,T,A |  |
|  |  | E |  |
|  |  | E |  |
|  |  |  |  |
| Student Learning Targets/Objectives (Students will know/Students will understand) |  |  |  |
| - Students will write conjunctions, disjunctions, and negations. <br> - Students will understand grouped statements. <br> - Students will change symbols into words. <br> - Students will write conditional and biconditional statements. <br> - Students will construt truth tables, including with negation, conditionals, biconditionals, and eight cases. <br> - Students will determine truth values without using a table. <br> - Students will understand self-contradiction and tautology. <br> - Students will determine if statements are logically equivalent. <br> - Students will apply DeMorgan's Law. <br> - Students will identify valid/invalid arguments with or without a truth table. <br> - Students will identify common fallacies. |  |  |  |
| Assessments (Pre, Formative, Summative, Other) |  | Denote required common assessments with an * |  |
| Tests* <br> Quizzes* |  |  |  |


| Oral Assessments <br> Homework <br> Projects <br> Journal |  |
| :---: | :---: |
| Teaching and Learning Activities |  |
| Activities | - Lecture <br> - Class discussion <br> - Cooperative Learning Activities <br> - Guided Practice <br> - Performance Assessments <br> - Projects <br> - HW Review <br> - Test Review <br> - Small Group Activities: <br> Use logic with circuit design <br> - Free circuit design software http://opencircuitdesign.com/ <br> - Link for activity http://acs.ist.psu.edu/discrete-math/ <br> Research paper on history of logic <br> Link for activity <br> http://www.maa.org/publications/periodicals/convergence/primary-historical-s ources-in-the-classroomdiscrete-mathematics-and-computer-science |
| Differentiation Strategies | Projects Cooperative Learning Structures <br> Simulation Activites Multiple Intelligence Activities <br> Technology Infusion Performance Assessments <br> Digital instruction Digital Assessment |
| Resources |  |
| - Textbook <br> - Websites: <br> Symbolic logic overview - http://www.youtube.com/watch?v=OLGVhszBlq4 <br> Statements, truth values and truth tables - http://www.math.csusb.edu/notes/logic/lognot/node1.html; http://www.math.csusb.edu/notes/quizzes/tablequiz/tablepractice.html <br> Logical equivalence and implication - http://www.math.csusb.edu/notes/logic/lognot/node2.html <br> - Calculators <br> - Technology: Powerpoint, Excel, etc. |  |

## Curriculum Format

| Content Area/ <br> Grade Level/ <br> Course: | Mathematics <br> 12 <br> Discrete Mathematics with Applications |
| :--- | :--- |
| Unit Plan Title: | Data Collection and Interpretation |
| Time Frame | 4 weeks |
| Anchor Standards/Domain* $\quad$ *i.e: ELA: reading, writing i.e.: Math: Algebra |  |

HSS-IC.A. Understand and evaluate random processes underlying statistical experiments
HSS-IC.B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies

## Unit Overview

The goal of many statistical studies is to show that changes in one variable cause changes in another variable. In this unit, we will look at why establishing caustion is so difficult and how designing an experiment using randomization and various methodologies (matched pairs and randomized block designs for example) can help to control bias and confounding. While in theory designing an experiement may seem plausible to students, data ethics and real life constraints such as time and money are factors which must be overcome.

## Standard Number(s) $\quad$ * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

Making Inferences and Justifying Conclusions

HSS.IC.A.1: Understand that statistics is a process for making inferences about population parameters based on a random sample from the population.
HSS.IC.B.6: Evaluate reports based on data

## MP: Standards for Math Practice

1. Make sense of problems and persevere in solving them.
2. Construct viable arguments and critique the reasoning of others.
3. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Attend to precision
6. Use appropriate tools strategically
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
and existing algorithms.
9.1.12.PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
9.4.12.Cl.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

## Intended Outcomes - \{Essential Questions\}

- How do we obtain data?
- To what extend is all data biased?
- To what extent does data collection methodology affect results?
- How can variables be eliminated through randomization?
- How does one decide between an observation study, an experiment and a simulation?
- To what extent can data be purposefully biased?


## Enduring Understandings

- Careful planning is essential to obtaining valid data
- Clairfying the question of interest leads to appropriate methodology
- The analysis is only as good as the data
- Students will understand how to deconstruct statistical information in an effort to evaluate its validity and assess the aims of the authors in presenting the information

In this unit plan, the following $21^{\text {st }}$ Century themes and skills are addressed.

|  | Check all that apply. 21 ${ }^{\text {st }}$ Century Themes | Indicate whether these skills are E-Encouraged, $\boldsymbol{T}$-Taught, or A-Assessed in this unit by marking $\mathbf{E}, \mathbf{T}, \mathbf{A}$ on the line before the appropriate skill. <br> 21 ${ }^{\text {st }}$ Century Skills |  |
| :---: | :---: | :---: | :---: |
| x | Global Awareness <br> Environmental Literacy <br> Health Literacy | E,A | Creativity and Innovation |
| x |  | E,T,A | Critical Thinking and Problem Solving |
| x |  | E | Communication |



## Wayne School District

 Curriculum Format| Content Area/ <br> Grade Level/ <br> Course: | Mathematics <br> 12 <br> Discrete Mathematics with Applications |
| :--- | :--- |
| Unit Plan Title: | Organizing and Summarizing Data |
| Time Frame | 4 weeks |
| Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| HSS-ID.A. Summarize, represent and interpret data on a single count or measurement variable <br> HSS-ID.B. Summarize, represent and interpret data on two categorical and quantitative variables |  | | Unit Overview |
| :--- |
| The volume of data available to us is overwhelming, yet easily organziable. In this unit, it is important to not only focus <br> on the skills and mechanics required to make graphical displays but rather to look out for overall patterns and trends <br> which can be used to make connections to the real world. Every set of data contains valuable context and even those <br> graphs which appear convincing may also be misleading. |

## Standard Number(s) * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

## Interpreting Categorical and Quantitative Data

HSS.ID.A.1: Represent data with plots on the real number line (dot plots, histograms, and boxplots)
HSS.ID.B.5: Summarize categorical data for two categories in a two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

## MP: Standards for Math Practice

1. Make sense of problems and persevere in solving them.
2. Construct viable arguments and critique the reasoning of others.
3. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Attend to precision
6. Use appropriate tools strategically
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning
8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.
8.1.12.DA.6: Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.
8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original and existing algorithms.
9.1.12. PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
9.4.12.Cl.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).
9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12 prof.CR3.a)
9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

## Intended Outcomes - \{Essential Questions\}

- What is the best way to organize qualitative or quantitative data?
- How can a graph be misleading?
- What should be kept in mind to create useful graphical displays?


## Enduring Understandings

- Graphical representations are the most common way that statistical information is represented
- One must be skeptical when looking at a graph or chart
- There is a proper technique to create charts and graphs that are not misleading

In this unit plan, the following $21^{\text {st }}$ Century themes and skills are addressed.


|  | Guided and Independent practice Collaborative Learning |  |
| :---: | :---: | :---: |
| Differentiation Strategies | Projects <br> Class activities <br> Multiple Intelligence Activities <br> Guided Practice <br> Performance Assessments <br> Digital instruction | Cooperative Learning Structures <br> Simulation Activites <br> Technology Infusion <br> Homework review <br> Quiz and Test review <br> Digital Assessment |
| Resources |  |  |
| - Textbook: A survey of Mathematics with Applications (Angel, Abbott, Runde); Pearson <br> - Supplemental Materials <br> - Graphing Calculator (TI-84) <br> - Computers <br> - Websites(Math open reference, chegg, Kahn Academy, Mathbits notebook, youtube educational) <br> - Power points, excel, google |  |  |


| Content Area/ Grade Level/ Course: | Mathematics <br> 12 <br> Discrete Mathematics with Applications |
| :---: | :---: |
| Unit Plan Title: | Numerically Summarizing Data |
| Time Frame | 5 weeks |
| Anchor Standards/Domain* *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| MA.9-12.S-ID: Interpreting Categorical and Quantitative Data MA.9-12.S-IC: Making Inferences and Justifying Conclusions |  |
| Unit Overview |  |
| Upon completion of this unit, students will be able to Summarize, represent and interpret data on a single count or mesurable variable. They will apply knowledge to make inferences and justify conclusions from sample surveys, experiments and observational studies. |  |
| Standard Number(s) * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4. |  |
|  |  |
| Intended Outcomes - \{Essential Questions\} |  |

- What is the best measure of central tendency given a set of data?
- How can the measure of spread be determined so that someone can tell how spread out the data is?
- How can the mean and standard deviation be calculated from gouped data (continuous quantitative)?
- How can a boxplot mimic the shape of the distribution?


## Enduring Understandings

- Even though a piece of data may not seem like it is an outlier, it very well may be.
- Know what the different measurements mean so when they are used, you know if the author used it to mislead data or was being truthful.
- Know what standard deviation means and how it is useful.

In this unit plan, the following $\mathbf{2 1}^{\text {st }}$ Century themes and skills are addressed.

|  | Check all that apply. <br> $21^{\text {st }}$ Century Themes | Indicate whether these skills are E-Encouraged, $\boldsymbol{T}$-Taught, or A-Assessed in this unit by marking $\mathbf{E}, \boldsymbol{T}, \mathbf{A}$ on the line before the appropriate skill. <br> 21 ${ }^{\text {st }}$ Century Skills |  |
| :---: | :---: | :---: | :---: |
| x | Global Awareness <br> Environmental Literacy <br> Health Literacy <br> Civic Literacy <br> Financial, Economic, Business, and <br> Entrepreneurial Literacy | ETA | Creativity and Innovation |
| x |  | ETA | Critical Thinking and Problem Solving |
| x |  | ETA | Communication |
| x |  | ETA | Collaboration |
| x |  |  |  |

## Student Learning Targets/Objectives (Students will know/Students will understand)

- Determine the arithmetic mean of a variable from raw data.
- Determine the median of a variable from raw data.
- Determine the mode of a variable fromm raw data.
- Determine the range of a variable from raw data.
- Determine the standard deviation of a variable from raw data.
- Determine the variance of a variable from raw data.
- Use the Empirical Rule to describe data that are bell shaped.
- Approximate the mean of a variable from grouped data.
- Compute the weighted mean.
- Approximate the standard deviation of a variable from grouped data.
- Determine and interpret z-scores.
- Interpret percentiles.
- Determine and interpret the interquartile range.
- Check a set of data for outliers.



## Wayne School District

 Curriculum Format| Content Area/ <br> Grade Level/ <br> Course: | Mathematics <br> 12 <br> Discrete Mathematics with Applications |
| :--- | :--- |
| Unit Plan Title: | Scatter Plots and Correlation |
| Time Frame | 3 weeks |
| Anchor Standards/Domain* $\quad$ *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| MA.9-12.S-ID: Interpreting Categorical and Quantitative Data <br> MA.9-12.S-CP: Conditional Probability and Rules of Probability |  |

## Unit Overview

Upon completion of this unit, students will be able to Summarize, represent and interpret data on a single count or mesurable variable. They will apply knowledge to interepret linear models. Students will also be able to understand independence and conditional probabilyt and use these to interpret data.

## Standard Number(s) * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

- HSS-ID.A. 1
- HSS-ID.C. 7
- HSS-ID.C. 8
- HSS-ID.C. 9
- HSS-CP.A. 4
- 8.1.12.DA. 5
- 8.1.12.DA. 6
- 8.1.12.AP. 1
- 9.1.12.PB. 2
- 9.2.12.CAP. 4
- 9.4.12.Cl. 1
- 9.4.12.CT. 2
- 9.4.12.IML. 3
- 9.4.12.TL. 1
- RST.11-12.3
- RST.9-10.7
$\square$
- RST.9-10.4


## Intended Outcomes - \{Essential Questions\}

- Is a regression model an effective model for prediction?
- Is there a difference between causation and correlation?
- Can causation be determined from correlation?


## Enduring Understandings

- Regression is an effective model for prediction.
- There is a difference between causation and correlation.

In this unit plan, the following $21^{\text {st }}$ Century themes and skills are addressed.


- Draw and interpret scatter diagrams.
- Describe the properties of the linear correlation coefficient.
- Compute and interpret the linear correlation coefficient.
- determine whether a linear relationship exists between two variables.
- Explain the difference between correlation and causation.
- Find the least-squares regression line and use the line to make predictions.
- Interpret the slope and the y-intercept of the least-squares regression line.
- Compute the sum of the squares residuals.

Assessments (Pre, Formative, Summative, Other) Denote required common assessments with an *
Lectures/Seminars
Question and answer (in class)
Oral Assessments
Homework and other written assignments
Digital assignments/assessments


- Textbook: A survey of Mathematics with Applications (Angel, Abbott, Runde); Pearson
- Supplemental Materials
- Graphing Calculator (TI-84)
- Computers
- Websites(Math open reference, chegg, Kahn Academy, Mathbits notebook, youtube educational)
- Power points, excel, google


## Wayne School District Curriculum Format

| Content Area/ | Mathematics |
| :--- | :--- |
| Grade Level/ | 12 |
| Course: | Discrete Mathematics and Applications |


| Unit Plan Title: | Probability |
| :--- | :--- | :--- |
| Time Frame | 6 weeks |
| Anchor Standards/Domain* $\quad$ *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| Usinf Probability to Make Decisions <br> HSS - MD.A. Calculate expected values and use them to solve problems. <br> HSS - MD.B. Use probability to evaluate outcomes of decisions. <br>  <br> Unit Overview <br> Random processes can be described mathematically by using a probability model: a list or description of the possible <br> outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a <br> number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a <br> probability model, sample points represent outcomes and combine to make up events: probabilities of events can be <br> computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an <br> understanding of independence and conditional probability, which can be approached through the analysis of <br> two-way tables. Technology plays an important role in probability by making it possible to simulate many possible <br> outcomes in a short amount of time. <br> Understanding independence and conditional probability is necessary to use them to interpret data. This will involve <br> the use of rules of probability to compute probabilities of compound events in a uniform probability model. To use <br> probability to make decisions, one must be able to calculate expected values and use them to solve problems as well <br> as be able to use probability to evaluate outcomes of decisions. |  |

## Standard Number(s) *i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

- HSS - MD.A.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space, graph the corresponding probability distribution using the same graphical displays as for data distributions.
- HSS - MD.A. 2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
- HSS - MD.A. 3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated, find the expected value.
- HSS - MD.A. 4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically, find the expected value.
- HSS - MD.B.5a, Find the expected payoff for a game of chance.
- HSS - MD.B.5b, Evaluate and compare strategies on the basis of expected values.
- 8.1.12.DA.5: Create data visualizations from large data sets to summarize, communicate, and
- support different interpretations of real-world phenomena.
- 8.1.12.DA.6: Create and refine computational models to better represent the relationships among
- different elements of data collected from a phenomenon or process.
- 8.1.12.AP.1: Design algorithms to solve computational problems using a combination of original
- and existing algorithms.
- 9.1.12. PB.2: Prioritize financial decisions by considering alternatives and possible consequences.
- 9.2.12.CAP.4: Evaluate different careers and develop various plans (e.g., costs of public, private, training schools) and timetables for achieving them, including educational/training requirements, costs, loans, and debt repayment.
- 9.4.12.Cl.1: Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.,


### 1.1.12prof.CR3a).

- 9.4.12.CT.2: Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E. 12 prof.CR3.a)
- 9.4.12.IML.3: Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)
- 9.4.12.TL.1: Assess digital tools based on features such as accessibility options, capacities, and utility for accomplishing a specified task (e.g., W.11-12.6.).
- RST.9-10.3./RST.11-12.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.
- RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
- RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

Intended Outcomes - \{Essential Questions\}

- What is probability?
- How can probability be simulated?
- What is a probability distribution?
- How does one determine probability for a given event?
- How are probabilities useful?
- Why is the computation of probabilities useful?
- How can permutations and combinations be used to calculate probabilities?


## Enduring Understandings

- Probability is a tool for measuring long term behavior.
- Probability is used to make inferences and predictions.
- Use permutations and combinations to solve real life situations.

In this unit plan, the following $\mathbf{2 1}^{\text {st }}$ Century themes and skills are addressed.
Indicate whether these skills are E-Encouraged, $\boldsymbol{T}$-Taught, or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill.

21 ${ }^{\text {st }}$ Century Skills
Check all that apply.
$21^{\text {st }}$ Century Themes
x

Global Awareness
E,
A

Creativity and Innovation


|  | Cooperative Learning Structures <br> Multiple Intelligence Activities <br> Performance Assessments |
| :--- | :--- |
| Honors |  |
| Resources |  |
| - Textbook: Angel, Allen, et al. A Survey of Mathematics with Applications, New York, Perason. 2013 |  |
| - calculators |  |
| - Technology: power point, excel, etc. |  |


| Content Area/ <br> Grade Level/ <br> Course: | Mathematics <br> 12 <br> Discrete Mathematics with Applications |
| :--- | :--- |
| Unit Plan Title: | Graph Theory |
| Time Frame | 3 weeks |
| Anchor Standards/Domain* $\quad$ *i.e: ELA: reading, writing i.e.: Math: Algebra |  |
| Mathematical Practice <br> MP: The Standards for Mathematical Practice describe varieties of expertise that mathematics education at all levels <br> should seek to develop in their students <br> MP 1, 2, 4, 6, 7 |  |
| Unit Overview |  |
| Mathematically proficient students can appkly the mathematics they know to solve problems arising in everyday life, <br> society, and the workplace. Those who can apply what they know are comfortable making assumptions and <br> approximations to simplify a complicated situation. They are able to identify important quantities in a practical <br> situation and map their relationship using graph theory tools such as diagrams, graphs, and flow charts. They can <br> analyze those relationships mathematically to draw conclusions. |  |

## Standard Number(s) * i.e: Math: F.LE.A. 4 i.e.: NJSLSA.R4.

- 9-12.G.MG. 3 Apply geometric concepts in modeling situations. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*
- 9-12.N.Q.A. 3 Reason quantitatively and use units to solve problems. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.*
- CC.K-12.MP. 4 Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
- K-12.MP. 5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler,


#### Abstract

a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.


## Intended Outcomes - \{Essential Questions\}

- What is graph theory and how is it applicable to real world situations?
- What are the different types of situations that can be simulated and solved with graph theory?


## Enduring Understandings

- Graph theory is a useful tool for solving real world problems.
- There are several aspects of graph theory that can help find paths and circuits that would otherwise take an extended period of time to find.

In this unit plan, the following $21^{\text {st }}$ Century themes and skills are addressed.

| Check all that apply. <br> 21 ${ }^{\text {st }}$ Century Themes |  | Indicate whether these skills are E-Encouraged, $\boldsymbol{T}$-Taught, or A-Assessed in this unit by marking E, T, A on the line before the appropriate skill. <br> $21^{\text {st }}$ Century Skills |  |
| :---: | :---: | :---: | :---: |
| x | Global Awareness <br> Environmental Literacy | EA | Creativity and Innovation <br> Critical Thinking and Problem Solving |
| x |  | ET |  |
|  | Health Literacy <br> Civic Literacy <br> Financial, Economic, Business, and Entrepreneurial Literacy | E | Communication Collaboration |
|  |  | E |  |
|  |  |  |  |

## Student Learning Targets/Objectives (Students will know/Students will understand)

- Students will understand the Konigsberg bridge problem.
- Students will know how to represent floor plans and maps.
- Students will know how to find a path, circuit, or bridge.
- Students will understand Euler's Theorem.
- Students will know how to determine Euler paths and circuits.

| - Students will understand Hamilton Circuits and how to determine the number of them. |  |
| :---: | :---: |
| Assessments (Pre, Formati | , Summative, Other) Denote required common assessments with an * |
| Tests* <br> Quizzes* <br> Class Discussion <br> Oral Assessments <br> Homework <br> Projects |  |
| Teaching and Learning Activities |  |
| Activities | Lectures <br> Class discussion <br> Cooperative Learning Activities <br> Guided Practice <br> Performance Assessments <br> Projects <br> Homework Review <br> Test review <br> Small group activities |
| Differentiation Strategies | Projects <br> Simulation Activities <br> Technology Infusion <br> Digital Instruction/ Assessment <br> Cooperative Learning Structures <br> Multiple Intelligence Activities <br> Performance Assessments |
| Honors |  |
| Resources |  |
| - Textbook: Angel, Allen, et al. A Survey of Mathematics with Applications, New York, Pearson 2013. <br> - calculators <br> - technology: power point, excel, etc. |  |

